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Lepton mixing angle $\theta_{13} = 0$ with a horizontal symmetry D_4

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Abstract: We discuss a model for the lepton sector based on the seesaw mechanism and on a D_4 family symmetry. The model predicts the mixing angle θ_{13} to vanish. The solar mixing angle θ_{12} is free — it will in general be large if one does not invoke finetuning. The model has an enlarged scalar sector with three Higgs doublets, together with two real scalar gauge singlets χ_i $(i = 1, 2)$ which have vacuum expectation values $\langle \chi_i \rangle_0$ at the seesaw scale. The atmospheric mixing angle θ_{23} is given by $\tan\theta_{23} = \langle \chi_2 \rangle_0 / \langle \chi_1 \rangle_0$, and it is maximal if the lagrangian is D_4-invariant; but D_4 may be broken softly, by a term of dimension two in the scalar potential, and then $\langle \chi_2 \rangle_0$ becomes different from $\langle \chi_1 \rangle_0$. Thus, the strength of the soft D_4 breaking controls the deviation of θ_{23} from $\pi/4$. The model predicts a normal neutrino mass spectrum $(m_3 > m_2 > m_1)$ and allows successful leptogenesis if $m_1 \sim 4 \times 10^{-3}\,\mathrm{eV}$; these properties of the model are independent of the presence and strength of the soft D_4 breaking.

Keywords: Discrete and Finite Symmetries, Beyond Standard Model, Neutrino Physics.

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1. Introduction

The idea of neutrino oscillations [1] through neutrino masses and lepton mixing [2] has proved successful and has led to continuous theoretical and experimental efforts and progress — for a review see, for instance, [3]. Still, no favoured theory for explaining the observed patterns of neutrino masses and lepton mixing has emerged yet; in recent years, myriads of textures and models for the lepton mass matrices have been proposed — for a review see, for instance, [4]. Only the smallness of neutrino masses has found a favourite explanation in the seesaw mechanism [5]. In this paper we employ that mechanism, and dismiss textures, trying instead to trace some features of the lepton sector to family *symmetries*. In particular, one may try and elucidate whether there is a connection between the smallness of the mixing angle θ_{13} and the maximality of the atmospheric mixing angle θ_{23}. It turns out that there is no such connection:

- In the model of [6], which is based on the non-abelian symmetry group $O(2)$ [7], and in the model of [8], which is based on the discrete symmetry D_4, one finds $\theta_{13} = 0$ and $\theta_{23} = \pi/4$; the model of [9], which is based on the permutation group S_3, displays only a minimal deviation from those predictions.
- The authors of [10], on the basis of a symmetry A_4, found $\theta_{23} = \pi/4$ but their θ_{13} is non-zero and can even be rather large.
- With the non-standard CP transformation used in [11] atmospheric mixing is maximal but θ_{13} is free.
- Conversely, it was shown in [12] that one can construct models based on abelian groups, in particular on $\mathbb{Z}_4$, which display $\theta_{13} = 0$ but have a free atmospheric mixing angle.

The listing above demonstrates that, without specifying the family symmetry group and other details of the model, no general statement can be made as regards a possible relation between θ_{13} and θ_{23}.

In this paper we want to discuss a modification of the D_4 model of [8]. In this modification one breaks the D_4 symmetry softly in such a way that $\theta_{13} = 0$ is preserved but the atmospheric mixing angle becomes *free* and controlled by the strength of the soft symmetry-breaking term. The model discussed in this paper has the same mixing features as the models in [12], but a completely different structure due to the *non*-abelian symmetry group. Both the original and the softly broken D_4 models belong to a class of models where the Yukawa-coupling matrices are diagonal, and lepton mixing originates exclusively in the Majorana mass matrix M_R of the heavy neutrinos; the *suppression of neutral flavour-changing interactions* is a general feature of that class of models [13]. We furthermore show that, in the softly broken D_4 model, successful leptogenesis can be traced back to the original D_4 model.

This paper is organized as follows. In section 2 we review the original D_4 model. The softly broken D_4 model is introduced in section 3, where we also investigate its predictions for lepton mixing. The neutrino mass spectrum, the effective mass in neutrinoless $\beta\beta$ decay, and leptogenesis are studied in section 4. In section 5 we perform a qualitative discussion of the renormalization-group corrections to the prediction $\theta_{13} = 0$. Our conclusions are found in section 6.

2. The original D_4 model

The D_4 model [8] has three lepton families, including three right-handed neutrinos which enable the seesaw mechanism [5]. We generically denote e, μ, and τ by α; we have then three left-handed lepton doublets $D_{\alpha L}$, three right-handed charged-lepton singlets α_R, and three right-handed neutrino singlets $\nu_{\alpha R}$. The scalar sector consists of three Higgs doublets ϕ_1, ϕ_2, and ϕ_3, together with two *real* gauge singlets χ_1 and χ_2.

The D_4 model has three symmetries of the $\mathbb{Z}_2$ type:

$$\begin{aligned} \mathbb{Z}_2^{(\tau)} &: D_{\tau L}\,,\ \tau_R\,,\ \nu_{\tau R}\,,\ \chi_2 \text{ change sign;} \\ \mathbb{Z}_2^{(\mathrm{tr})} &: D_{\mu L} \leftrightarrow D_{\tau L}\,,\ \mu_R \leftrightarrow \tau_R\,,\ \nu_{\mu R} \leftrightarrow \nu_{\tau R}\,,\ \chi_1 \leftrightarrow \chi_2\,,\ \phi_3 \to -\phi_3\,; \\ \mathbb{Z}_2^{(\mathrm{aux})} &: \nu_{eR}\,,\ \nu_{\mu R}\,,\ \nu_{\tau R}\,,\ \phi_1,\ e_R \text{ change sign.} \end{aligned} \tag{2.1}$$

The symmetry $\mathbb{Z}_2^{(\tau)}$ flips the sign of all multiplets of the τ family, while $\mathbb{Z}_2^{(\mathrm{tr})}$ exchanges the multiplets of the μ and τ families. The auxiliary symmetry $\mathbb{Z}_2^{(\mathrm{aux})}$ ensures that ϕ_2 and ϕ_3 do not have Yukawa couplings to the $\nu_{\alpha R}$, so that the spontaneous breaking of $\mathbb{Z}_2^{(\mathrm{tr})}$, which is necessary for $m_\mu \neq m_\tau$, does not have consequences in the neutrino Dirac mass matrix M_D.

As discussed in [8], $\mathbb{Z}_2^{(\tau)}$ and $\mathbb{Z}_2^{(\mathrm{tr})}$ do not commute, and together they generate a non-abelian group D_4 with eight elements. This group has five inequivalent irreducible representations (irreps): one two-dimensional irrep and four one-dimensional irreps. It is clear that $(D_{\mu L}, D_{\tau L})$, (μ_R, τ_R), $(\nu_{\mu R}, \nu_{\tau R})$, and (χ_1, χ_2) transform as doublets of D_4, whereas ϕ_1 and ϕ_2 transform according to the trivial one-dimensional irrep, and ϕ_3 according to a non-trivial one-dimensional irrep.

The above multiplets and symmetries determine the Yukawa lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} = & - \left[y_1 \bar{D}_{eL} \nu_{eR} + y_2 \left(\bar{D}_{\mu L} \nu_{\mu R} + \bar{D}_{\tau L} \nu_{\tau R}\right)\right] \tilde{\phi}_1 - \\ & - y_3 \bar{D}_{eL} e_R \phi_1 - y_4 \left(\bar{D}_{\mu L} \mu_R + \bar{D}_{\tau L} \tau_R\right) \phi_2 - y_5 \left(\bar{D}_{\mu L} \mu_R - \bar{D}_{\tau L} \tau_R\right) \phi_3 + \\ & + \frac{1}{2} y_\chi \nu_{eR}^T C^{-1} \left(\nu_{\mu R} \chi_1 + \nu_{\tau R} \chi_2\right) + \text{H.c.}\,, \end{aligned} \tag{2.2}$$

where $\tilde{\phi}_1 \equiv i\tau_2 \phi_1^*$; they also determine the Majorana mass terms

$$\mathcal{L}_{\mathrm{M}} = \frac{1}{2} \left[M^* \nu_{eR}^T C^{-1} \nu_{eR} + M'^* \left(\nu_{\mu R}^T C^{-1} \nu_{\mu R} + \nu_{\tau R}^T C^{-1} \nu_{\tau R}\right)\right] + \text{H.c.} \tag{2.3}$$

The vacuum expectation values (VEVs) $\langle 0|\phi_j^0|0\rangle = v_j/\sqrt{2}$ $(j = 1,2,3)$ fix the charged-lepton masses as

$$\sqrt{2} m_e = |y_3 v_1|\,, \tag{2.4}$$

$$\sqrt{2} m_\mu = |y_4 v_2 + y_5 v_3|\,, \tag{2.5}$$

$$\sqrt{2} m_\tau = |y_4 v_2 - y_5 v_3|\,. \tag{2.6}$$

The neutrino Dirac mass matrix is given by

$$M_D = \mathrm{diag}\,(a, b, b)\,, \tag{2.7}$$

with $\sqrt{2}a = y_1^* v_1$ and $\sqrt{2}b = y_2^* v_1$. The charged-lepton mass matrix and M_D are *simultaneously* diagonal. Therefore, lepton mixing must result exclusively from a non-diagonal M_R, and this arises by virtue of the non-zero VEVs of $\chi_{1,2}$. Let us write those VEVs as

$$\begin{aligned}\langle 0\,|\chi_1|\,0\rangle &= W \cos\gamma\,, \\ \langle 0\,|\chi_2|\,0\rangle &= W \sin\gamma\,, \end{aligned} \tag{2.8}$$

with W real and positive. Then the Majorana mass matrix for the right-handed neutrino singlets is

$$M_R\,(\gamma) = \begin{pmatrix} M & M_\chi \cos\gamma & M_\chi \sin\gamma \\ M_\chi \cos\gamma & M' & 0 \\ M_\chi \sin\gamma & 0 & M' \end{pmatrix}, \tag{2.9}$$

where $M_\chi = y_\chi^* W$. The effective Majorana mass matrix for the light neutrinos is given by

$$\mathcal{M}_\nu\,(\gamma) = -M_D^T\, M_R\,(\gamma)^{-1} M_D\,, \tag{2.10}$$

where $M_D = M_D^T$ is in equation (2.7). Since the charged-lepton mass matrix is diagonal, the lepton mixing matrix U is found simply through the diagonalization procedure

$$U^T \mathcal{M}_\nu\,(\gamma)\, U = \mathrm{diag}\,(m_1, m_2, m_3)\,, \tag{2.11}$$

with real and non-negative masses m_j $(j = 1,2,3)$ for the light neutrinos.

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In the D_4-invariant scalar potential $V_{\rm sym}$, the χ-dependent terms are [8]

$$V_{\rm sym} = \cdots + \left(\chi_1^2 + \chi_2^2\right) \left[-\mu + \rho_1 \phi_1^\dagger \phi_1 + \rho_2 \left(\phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3\right)\right] + \lambda \left(\chi_1^2 + \chi_2^2\right)^2 + \\ + \left(\chi_1^2 - \chi_2^2\right) \left(\eta \phi_2^\dagger \phi_3 + \eta^* \phi_3^\dagger \phi_2\right) + \lambda' \left(\chi_1^2 - \chi_2^2\right)^2 , \tag{2.12}$$

where μ, ρ_1, ρ_2, λ, η, and λ' are c-numbers. Then, γ is determined by the minimization of

$$f_{\rm sym}(\gamma) = \lambda' W^4 \cos^2 2\gamma + {\rm Re}\left(\eta v_2^* v_3\right) W^2 \cos 2\gamma\, . \tag{2.13}$$

Provided $\lambda' > 0$ and $|{\rm Re}(\eta v_2^* v_3)| \leq 2\lambda' W^2$, the minimum of f is at

$$\cos 2\gamma = -\frac{{\rm Re}\left(\eta v_2^* v_3\right)}{2\lambda' W^2}\, . \tag{2.14}$$

According to the seesaw mechanism [5], we assume

$$|M|\, , |M'| \gg v\, , \tag{2.15}$$

where $v = \sqrt{|v_1|^2 + |v_2|^2 + |v_3|^2} \simeq 246\,{\rm GeV}$. We furthermore assume $W \sim |M|, |M'|$, so that the off-diagonal matrix elements of $M_R(\gamma)$ are not much smaller than the diagonal matrix elements, lest the solar mixing angle becomes very small. Under these assumptions, $|\cos 2\gamma|$ is negligibly small and, for all practical purposes,

$$\langle 0\, |\chi_1|\, 0\rangle = \langle 0\, |\chi_2|\, 0\rangle = \frac{W}{\sqrt{2}}\, , \tag{2.16}$$

or $\gamma = \pi/4$. Corrections to this value of γ are of order v^2/W^2.

One can easily check that [14]

$$S M_R\left(\frac{\pi}{4}\right) S = M_R\left(\frac{\pi}{4}\right), \tag{2.17}$$

where

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{2.18}$$

Then, due to equations (2.10) and (2.7), also $S \mathcal{M}_\nu(\pi/4)\, S = \mathcal{M}_\nu(\pi/4)$. From this the predictions of the D_4 model follow [8]: maximal atmospheric mixing angle, i.e. $\theta_{23} = \pi/4$; free, in general large, solar mixing angle θ_{12}; and $\theta_{13} = 0$. One also finds — see Section 4 — that, because the (μ, τ) matrix element of $M_R(\gamma)$ vanishes, the inverted neutrino mass spectrum is excluded.

3. The softly broken D_4 model

In the original D_4 model of the previous section, all three $\mathbb{Z}_2$ symmetries of equation (2.1) are spontaneously broken, and none of them is broken in the lagrangian itself. In this section we introduce a soft dimension-two breaking of $\mathbb{Z}_2^{(\rm tr)}$. It is easily found that there are two terms performing that breaking: one of them is $\phi_2^\dagger \phi_3$ plus its hermitean conjugate, which is largely irrelevant; the other one is

$$V_{\text{soft}} = \mu_{\text{soft}} \left(\chi_1^2 - \chi_2^2 \right) , \tag{3.1}$$

which is to be added to the scalar potential. Now the angle γ of equation (2.8) is determined by the minimization of

$$f(\gamma) = f_{\text{sym}}(\gamma) + \mu_{\text{soft}} W^2 \cos 2\gamma \, , \tag{3.2}$$

leading to

$$\cos 2\gamma = -\frac{\mu_{\text{soft}} + \mathrm{Re}\,(\eta v_2^* v_3)}{2\lambda' W^2} \, . \tag{3.3}$$

We assume $|\mu_{\text{soft}}|$ to be of the same order of magnitude as W, i.e. we assume it to be of the seesaw scale; this results in a non-negligible deviation of γ from $\pi/4$. As discussed in the previous section, such a deviation is in general present, cf. equation (2.14), yet without the soft breaking in equation (3.1) it is very small because $v \ll W$.[1]

Defining

$$S(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\gamma & \sin 2\gamma \\ 0 & \sin 2\gamma & -\cos 2\gamma \end{pmatrix} = S(\gamma)^{-1} \, , \tag{3.4}$$

which generalizes the matrix S in equation (2.18), it is easy to check that

$$S(\gamma)\, M_R(\gamma)\, S(\gamma) = M_R(\gamma) \ \Rightarrow \ S(\gamma)\, \mathcal{M}_\nu(\gamma)\, S(\gamma) = \mathcal{M}_\nu(\gamma) \, . \tag{3.5}$$

The orthogonal matrix $S(\gamma)$ has a unique eigenvalue -1, corresponding to the eigenvector

$$u_3 = \begin{pmatrix} 0 \\ -\sin\gamma \\ \cos\gamma \end{pmatrix} . \tag{3.6}$$

Equation (3.5) then implies that u_3 is also an eigenvector of $\mathcal{M}_\nu(\gamma)$. Since u_3 is real and its first entry is zero, this eigenvector — possibly multiplied by a phase — must constitute the third column of the lepton mixing matrix U. Explicit calculation of $\mathcal{M}_\nu(\gamma)$ using

$$\begin{aligned} M_R(\gamma)^{-1} &= \frac{1}{M' \left(M M' - M_\chi^2 \right)} \times \\ &\times \begin{pmatrix} M'^2 & -M' M_\chi \cos\gamma & -M' M_\chi \sin\gamma \\ -M' M_\chi \cos\gamma & M M' - M_\chi^2 \sin^2\gamma & M_\chi^2 \sin\gamma\cos\gamma \\ -M' M_\chi \sin\gamma & M_\chi^2 \sin\gamma\cos\gamma & M M' - M_\chi^2 \cos^2\gamma \end{pmatrix} \end{aligned} \tag{3.7}$$

shows that the eigenvalue is $-b^2/M'$, hence the mass of the third light neutrino is

$$m_3 = \left| \frac{b^2}{M'} \right| . \tag{3.8}$$

[1]We might as well assume W to be of the Fermi scale, and then μ_{soft} would be unnecessary. But this would imply the off-diagonal elements of M_R being much smaller than the diagonal ones, and this would prevent the fitting of the observed large solar mixing angle.

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Thus,

$$U_{e3} = 0 \tag{3.9}$$

is *exact* at the tree level, while

$$\sin^2 2\theta_{\rm atm} = 4 \left|U_{\mu 3}\right|^2 \left(1 - \left|U_{\mu 3}\right|^2\right) = \sin^2 2\gamma\,, \tag{3.10}$$

i.e. $\theta_{\rm atm} = \gamma$. The atmospheric mixing angle is equal to the angle γ from the VEVs of $\chi_{1,2}$ in equations (2.8).

We thus have a *model* where the atmospheric mixing angle is arbitrary, since its deviation from $\pi/4$ is controlled by the strength $\mu_{\rm soft}$ of the soft-breaking term $V_{\rm soft}$ in equation (3.1); on the other hand — at least at the tree level — the angle θ_{13} vanishes. The solar mixing angle is completely arbitrary, but in general it will be large.

We have broken the μ–τ interchange symmetry $\mathbb{Z}_2^{(\rm tr)}$ softly in the scalar potential by means of the term in equation (3.1). That soft-breaking term is of dimension two. One might, in addition, also break $\mathbb{Z}_2^{(\rm tr)}$ through terms of dimension three, viz. by assuming $(M_R)_{\mu\mu}$ to be different from $(M_R)_{\tau\tau}$. A general mixing matrix U would then follow. It is, however, consistent to avoid soft-breaking terms of dimension three while allowing those of dimension two.

4. Neutrino masses and leptogenesis

The diagonalization of the mass matrices in equations (2.9) and (2.10) can be related to the diagonalization of those matrices when $\gamma = \pi/4$. Let us define the orthogonal matrix

$$O = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (\cos\gamma + \sin\gamma)/\sqrt{2} & (\cos\gamma - \sin\gamma)/\sqrt{2} \\ 0 & (\sin\gamma - \cos\gamma)/\sqrt{2} & (\cos\gamma + \sin\gamma)/\sqrt{2} \end{pmatrix} \tag{4.1}$$

Then we easily find that

$$O^T M_R(\gamma)\, O = M_R\left(\frac{\pi}{4}\right) \;\Rightarrow\; O^T \mathcal{M}_\nu(\gamma)\, O = \mathcal{M}_\nu\left(\frac{\pi}{4}\right). \tag{4.2}$$

The matrix

$$U' = e^{i\hat\alpha} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} e^{i\hat\beta}, \tag{4.3}$$

where

$$e^{i\hat\alpha} = \mathrm{diag}\left(1, e^{i\alpha}, e^{i\alpha}\right), \tag{4.4}$$

$c_{12} \equiv \cos\theta_{12}$, and $s_{12} \equiv \sin\theta_{12}$, diagonalizes $\mathcal{M}_\nu(\pi/4)$ [8]. The matrices $e^{i\hat\alpha}$ and $e^{i\hat\beta}$ are diagonal phase matrices; the first of them is unphysical, and it can be shown that it is of the form given in equation (4.4). According to equation (4.2), the matrix U which diagonalizes $\mathcal{M}_\nu(\gamma)$ is

$$U = O\,U' = e^{i\hat\alpha} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}\cos\gamma & c_{12}\cos\gamma & -\sin\gamma \\ -s_{12}\sin\gamma & c_{12}\sin\gamma & \cos\gamma \end{pmatrix} e^{i\hat\beta}, \tag{4.5}$$

cf. equation (3.6). With $e^{i\hat\beta} = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$, the physical Majorana phases can be chosen to be $\Delta \equiv 2(\beta_1 - \beta_2)$ and $2(\beta_1 - \beta_3)$.

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A salient feature of the D_4 model — whether softly broken or not — is the fact that $(M_R)_{\mu\tau} = 0$. This translates into [8]

$$
\begin{aligned}
0 &= \sum_{j=1}^{3} m_j^{-1} U_{\mu j} U_{\tau j} \\
&= e^{2i\alpha} \sin\gamma \cos\gamma \left(\frac{s_{12}^2 e^{2i\beta_1}}{m_1} + \frac{c_{12}^2 e^{2i\beta_2}}{m_2} - \frac{e^{2i\beta_3}}{m_3} \right) = 0\,.
\end{aligned} \tag{4.6}
$$

We see that the expression within parentheses is zero irrespective of the value of γ. Hence, the effective mass probed in neutrinoless $\beta\beta$ decay, $|\langle m \rangle| = |(\mathcal{M}_\nu)_{ee}|$, is, just as in [8],

$$
|\langle m \rangle| = \left| m_1 c_{12}^2 e^{-2i\beta_1} + m_2 s_{12}^2 e^{-2i\beta_2} \right| = \frac{m_1 m_2}{m_3}\,. \tag{4.7}
$$

Moreover, just as in the original D_4 model, only the normal spectrum $m_1 < m_2 < m_3$ is allowed. ($m_1 < m_2$ holds by definition, and the solar mass-squared difference is $\Delta m^2_\odot = m_2^2 - m_1^2$.)

For baryogenesis via leptogenesis [18] — for reviews, see for instance [19] — we must diagonalize the mass matrix M_R. Denoting the unitary diagonalization matrix by V and the heavy-neutrino masses by M_j, one has

$$
V^T M_R(\gamma)\, V = V'^T M_R\left(\frac{\pi}{4}\right) V' = \mathrm{diag}\,(M_1,\, M_2,\, M_3)\,, \tag{4.8}
$$

with $V = O\,V'$. The matrix which enters the calculation of the CP asymmetry generated by the decay of the heavy neutrinos is given by [19]

$$
R = V^T M_D M_D^\dagger V^* = V'^T M_D M_D^\dagger V'^*\,. \tag{4.9}
$$

The second equality in equation (4.9) is justified by the twofold degeneracy of M_D, see equation (2.7). Thus, in the softly broken D_4 model only V' is relevant for leptogenesis. As emphasized before, U' and V' coincide with the diagonalization matrices of the original D_4 model. Hence, the soft breaking of D_4 *does not alter* the expression of the lepton asymmetry derived in [20], and one obtains the same results irrespective of the value of $\gamma = \theta_{23}$. The masses of the third-generation neutrinos, both in the light- and heavy-neutrino sectors (i.e., both m_3 and M_3), do not occur in the expression for leptogenesis — which depends only on $m_{1,2}$, $M_{1,2}$, θ_{12}, and Δ [20, 21]. However, m_3 enters leptogenesis indirectly, because the Majorana phase Δ may be expressed as a function of m_3; indeed, from equation (4.6) [20],

$$
\left| m_2 s_{12}^2 e^{i\Delta} + m_1 c_{12}^2 \right| = \frac{m_1 m_2}{m_3}\,, \tag{4.10}
$$

hence

$$
\cos\Delta = \frac{(m_1 m_2/m_3)^2 - c_{12}^4 m_1^2 - s_{12}^4 m_2^2}{2 c_{12}^2 s_{12}^2 m_1 m_2}\,. \tag{4.11}
$$

Successful leptogenesis in the D_4 model requires [20] the lightest-neutrino mass m_1 to be below 10^{-2} eV. In that region equation (4.11) gives a strong restriction on the allowed range for m_1, since $\cos\Delta$ must be larger than -1; using $\theta_{12} = 33^\circ$, $\Delta m^2_\odot = 7.1 \times 10^{-5}\,\mathrm{eV}^2$, and

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$\Delta m^2_{\mathrm{atm}} = m_3^2 - m_1^2 = 2 \times 10^{-3}\,\mathrm{eV}^2$ [3], one obtains $2.9 \times 10^{-3}\,\mathrm{eV} \lesssim m_1 \lesssim 7.1 \times 10^{-3}\,\mathrm{eV}$. In order to reproduce the baryon-over-photon ratio η_B, M_1 must lie in between 10^{11} and $10^{12}\,\mathrm{GeV}$, if we assume $M_2 \gg M_1$. The maximum value of η_B is attained for $m_1 \simeq 4 \times 10^{-3}\,\mathrm{eV}$, and the experimental value $\eta_B \sim 6.5 \times 10^{10}$ [22] can easily be reproduced — see figure 1 of [21].

5. Radiative corrections

The form of $\mathcal{M}_\nu$ given by equations (2.7), (2.9), and (2.10) holds only at the seesaw scale m_R. Radiative corrections must be taken into account if one wants to calculate $\mathcal{M}_\nu$ at the electroweak scale. The effective operators relevant in this context are [15, 16, 17]

$$\begin{aligned}\mathcal{O}_{ij} &= \sum_{\alpha,\beta} \sum_{a,b,c,d} \left[(D_{\alpha L})_a^T \, \kappa^{(ij)}_{\alpha\beta} C^{-1} \left(D_{\beta L}\right)_c \right] \left[\varepsilon^{ab} \left(\phi_i\right)_b \right] \left[\varepsilon^{cd} \left(\phi_j\right)_d \right] &(5.1)\\ &= \sum_{\alpha,\beta} \kappa^{(ij)}_{\alpha\beta} \left(\nu^T_{\alpha L} \phi^0_i - \alpha^T_L \phi^+_i \right) C^{-1} \left(\nu_{\beta L} \phi^0_j - \beta_L \phi^+_j \right), &(5.2)\end{aligned}$$

where a, b, c, d are SU(2) indices, $\varepsilon = i\tau_2$ is the 2×2 antisymmetric tensor, and the $\kappa^{(ij)}$ are matrices in family space. Note that we may, without loss of generality, enforce the condition $\kappa^{(ij)}_{\alpha\beta} = \kappa^{(ji)}_{\beta\alpha}$. We want to discuss the renormalization-group (RG) evolution of the effective coupling matrices $\kappa^{(ij)}$ from the seesaw scale m_R down to the electroweak scale; the latter may be represented by the Z^0 mass m_Z. Our aim is to estimate a possible modification of equation (3.9) by the RG evolution. We denote the dimensionless variable of the RG by t; the RG evolution goes from $t_0 = \ln\left(m_R/m_Z\right)$ to $t_1 = 0$. The initial conditions for the RG equations are

$$\begin{aligned}\kappa^{(11)}\left(t_0\right) &= \frac{1}{v_1^2}\, \mathcal{M}_\nu\left(\gamma\right),\\ \kappa^{(ij)}\left(t_0\right) &= 0 \text{ for all other } (ij). &(5.3)\end{aligned}$$

In equation (5.3) we have used the fact that in our model only the Higgs doublet ϕ_1 has Yukawa couplings to the right-handed neutrinos, and gives thereby rise to M_D.

The symmetry $\mathbb{Z}_2^{(\mathrm{aux})}$, which is preserved throughout the RG evolution since it is only broken spontaneously at the electroweak scale, implies that there is a symmetry $\phi_1 \to -\phi_1$ in the evolution of the operators in equation (5.2). Since at the high scale only $\mathcal{O}_{11}$ is present, which is invariant under $\mathbb{Z}_2^{(\mathrm{aux})}$, it follows that the operators $\mathcal{O}_{12}$ and $\mathcal{O}_{13}$, which are odd under $\mathbb{Z}_2^{(\mathrm{aux})}$, remain zero at all scales. At the electroweak scale we will then have

$$\mathcal{M}_\nu = \sum_{i=1}^{3} v_i^2 \kappa^{(ii)}\left(t_1\right) + v_2 v_3 \left[\kappa^{(23)}\left(t_1\right) + \kappa^{(32)}\left(t_1\right) \right]. \tag{5.4}$$

The matrix $\mathcal{M}_\nu$ at the electroweak scale will not in general yield $U_{e3} = 0$. Indeed, using $U^T \mathcal{M}_\nu U = \mathrm{diag}\left(m_1, m_2, m_3\right)$, we find that $U_{e3} = 0$ if and only if there is a vector u with a zero first entry such that $\mathcal{M}_\nu u = z u^*$, where z is a complex number and $m_3 = |z|$.

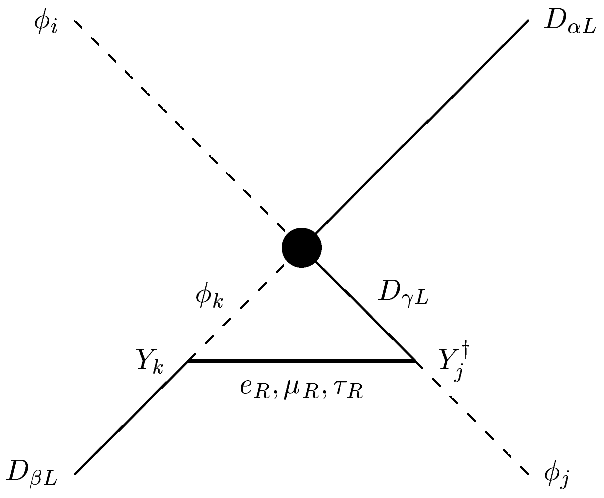

Figure 1: A typical vertex correction which introduces flavour dependence in the renormalization of the operators $\mathcal{O}_{ij}$ of equation (5.2). The relevant Yukawa-coupling matrices are indicated.

This means that the symmetric matrix $\mathcal{M}_\nu$ satisfies

$$\left[\left|(\mathcal{M}_\nu)_{e\mu}\right|^2 - \left|(\mathcal{M}_\nu)_{e\tau}\right|^2\right](\mathcal{M}_\nu)_{\mu\tau} = (\mathcal{M}_\nu)^*_{e\mu}(\mathcal{M}_\nu)_{e\tau}(\mathcal{M}_\nu)_{\mu\mu} - (\mathcal{M}_\nu)_{e\mu}(\mathcal{M}_\nu)^*_{e\tau}(\mathcal{M}_\nu)_{\tau\tau}\,. \tag{5.5}$$

This condition will in general not be satisfied by the matrix $\mathcal{M}_\nu$ at the electroweak scale given in equation (5.4), but it *is* satisfied by the matrix $\mathcal{M}_\nu$ at the seesaw scale given by equations (2.7), (2.9), and (2.10).

Denoting the set of all matrices $\kappa^{(ij)}$ by κ, the differential equation for their RG evolution can symbolically be written as

$$16\pi^2\,\frac{\mathrm{d}\kappa(t)}{\mathrm{d}t} = L_t\left[\kappa(t)\right], \tag{5.6}$$

where L_t is a linear operator acting on the $\kappa^{(ij)}$. In L_t, the Yukawa and gauge coupling constants appear in second order and the quartic Higgs couplings in first order. The solution of equation (5.6) is formally given by the series

$$\kappa(t) = \kappa(t_0) + \frac{1}{16\pi^2}\int_{t_0}^{t}\mathrm{d}t' L_{t'}\left[\kappa(t_0)\right] + \left(\frac{1}{16\pi^2}\right)^2\int_{t_0}^{t}\mathrm{d}t'\int_{t_0}^{t'}\mathrm{d}t'' L_{t'}\left[L_{t''}\left[\kappa(t_0)\right]\right] + \cdots\,. \tag{5.7}$$

Flavour dependence in $\dot{\kappa}^{(ij)}$ (the dot denotes the derivative relative to t) can only be introduced by the Yukawa couplings. One can easily convince oneself that only vertex corrections of the type depicted in figure 1 need a closer look at.[2] The Yukawa-coupling matrices of the Higgs doublets ϕ_j to the right-handed charged leptons are given by the

[2] All the types of graphs relevant for the computation of the full RG equations for the $\kappa^{(ij)}$ are depicted in [15, 17].

second line of the Yukawa lagrangian in equation (2.2); they are

$$\begin{aligned} Y_1 &= \mathrm{diag}\,(y_3^*, 0, 0)\,, \\ Y_2 &= \mathrm{diag}\,(0, y_4^*, y_4^*)\,, \\ Y_3 &= \mathrm{diag}\,(0, y_5^*, -y_5^*)\,. \end{aligned} \tag{5.8}$$

One finds from figure 1 that $\dot{\kappa}^{(ij)} + \dot{\kappa}^{(ji)}$ obtains a contribution $\sum_k \left[\kappa^{(ik)} Y_j^\dagger Y_k + \kappa^{(jk)} Y_i^\dagger Y_k\right]$ plus the transposed term.

A non-zero $\kappa^{(23)}$ is induced from $\kappa^{(11)}(t_0)$ in two steps. Firstly, vertex corrections from the quartic terms

$$\lambda' \left(\phi_1^\dagger \phi_2\right)^2 + \lambda'' \left(\phi_1^\dagger \phi_3\right)^2 \tag{5.9}$$

in the Higgs potential induce non-zero matrices $\kappa^{(22)}$ and $\kappa^{(33)}$ via

$$16\pi^2\, \frac{\mathrm{d}\kappa^{(22)}}{\mathrm{d}t} \sim \lambda' \kappa^{(11)}\,, \tag{5.10}$$

$$16\pi^2\, \frac{\mathrm{d}\kappa^{(33)}}{\mathrm{d}t} \sim \lambda'' \kappa^{(11)}\,. \tag{5.11}$$

Secondly, from a vertex correction of the type in figure 1 one obtains

$$16\pi^2\, \frac{\mathrm{d}}{\mathrm{d}t} \left[\kappa^{(23)} + \kappa^{(32)}\right] \sim \kappa^{(22)} Y_3^\dagger Y_2 + Y_2^T Y_3^* \kappa^{(22)} + \kappa^{(33)} Y_2^\dagger Y_3 + Y_3^T Y_2^* \kappa^{(33)}\,. \tag{5.12}$$

Equations (5.10), (5.11), and (5.12) tell us that, in the expansion of equation (5.7), the matrices $\kappa^{(23)}$ and $\kappa^{(32)}$ appear first at second order. Since

$$Y_3^\dagger Y_2 = \left(Y_2^\dagger Y_3\right)^\dagger = y_4^* y_5\, \mathrm{diag}\,(0, +1, -1)\,, \tag{5.13}$$

the vector u_3 of equation (3.6) will not be eigenvector of $\kappa^{(23)} + \kappa^{(32)}$. One can check that, at second order in the expansion of equation (5.7), u_3 is still an eigenvector of the $\kappa^{(ii)}$ ($i = 1, 2, 3$). Therefore, we estimate that the terms in $\mathcal{M}_\nu$ which are responsible for $U_{e3} \neq 0$ will typically be suppressed by a factor of the order of

$$\frac{1}{(16\pi^2)^2} \left(\ln \frac{m_R}{m_Z}\right)^2 \left(\frac{m_\tau}{|v_2|}\right)^2 \left|\lambda^{\prime,\prime\prime}\right|. \tag{5.14}$$

In this equation we have taken into account that there are two integrations over the interval of length t_0 and that the relevant Yukawa couplings are of the order $m_\tau/|v_2|$. Numerically, using $t_0 \sim 10$ and even if we take the quartic Higgs couplings to be of order one, the suppression factor is at least 10^{-4}. Therefore, to a good approximation we will still have $U_{e3} = 0$ at the electroweak scale.[3]

[3] The details of the RGE for multiple-Higgs-doublet models will be published elsewhere.

6. Conclusions

In this paper we have modified the D_4 model of [8] by allowing a soft breaking of the subgroup $\mathbb{Z}_2^{(\mathrm{tr})}$ of D_4, i.e. by breaking the μ–τ exchange symmetry softly — see equation (2.1). An essential ingredient of the D_4 model is the D_4 doublet (χ_1, χ_2) of real scalar gauge singlets, with VEVs and masses of the seesaw scale m_R. It is the VEVs of those scalars which induce lepton mixing. In particular, one has

$$\tan\theta_{23} = \frac{\langle\chi_2\rangle_0}{\langle\chi_1\rangle_0}\,. \tag{6.1}$$

Requiring the soft breaking of $\mathbb{Z}_2^{(\mathrm{tr})}$ to occur exclusively through terms of dimension two in the lagrangian, we find that the term of equation (3.1) appears in the scalar potential. Whereas without soft breaking the VEVs of χ_1 and χ_2 are (almost) equal, the soft breaking of $\mathbb{Z}_2^{(\mathrm{tr})}$ induces a deviation from this equality. Thus, the strength of the soft-breaking term in equation (3.1) determines the deviation of θ_{23} from 45°, small deviations being natural in a technical sense.

We have demonstrated that the soft D_4 breaking has no effect on the neutrino mass spectrum, on the effective mass $|\langle m\rangle|$ in neutrinoless $\beta\beta$ decay, and on leptogenesis. Just as in the original D_4 model, we predict a normal spectrum $m_1 < m_2 < m_3$, $|\langle m\rangle| < 10^{-2}\,\mathrm{eV}$, and we find that leptogenesis is successful when $m_1 \sim 4\times10^{-3}\,\mathrm{eV}$.

Like in all multi-Higgs models where the Yukawa-coupling matrices are diagonal and lepton mixing originates solely in M_R, neutral flavour-changing interactions are suppressed [13]: on the one hand, the amplitudes for e.g. $\mu^- \to e^-\gamma$ and $Z^0 \to e^+\mu^-$ are proportional to m_R^{-2}, hence highly suppressed; on the other hand, a decay like $\mu^- \to e^-e^+e^-$ is suppressed by Yukawa couplings but might fall in the discovery range of forthcoming experiments.

The D_4 model — whether softly broken or not — predicts $\theta_{13} = 0$. We have estimated, through renormalization-group methods, the radiative corrections to this relation to yield $\theta_{13} \sim 10^{-4}$ or smaller — the same estimate also applies to the $\mathbb{Z}_2$ model of [6]. The solar mixing angle is not predicted in the D_4 model; without finetuning it will be large.

In summary, we have constructed an extension of the Standard Model based on the non-abelian family symmetry group D_4. Our model features a vanishing θ_{13} together with non-maximal atmospheric mixing. The model is in agreement with all the existing data on neutrinos and allows successful leptogenesis.

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